Acoustic Scattering Computations for High-Speed Rotors using Rotating Reference Frames

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ABSTRACT

The rotational effect is a crucial factor that needs to be considered when dealing with acoustic wave scattering through high-speed rotors, e.g., in propulsion line simulations. Neglecting the rotational effect can lead to inaccurate results, especially in scenarios such as computing the transmission loss through compressors. Therefore, it is essential to incorporate the rotational effect in the governing linearized equations for fluid flow. This approach involves utilizing a rotating frame of reference in a stationary geometry, which has been demonstrated to be both straightforward and numerically inexpensive. Overall, the findings of this study highlight the importance of considering rotational effects in acoustic wave scattering simulations, particularly in high-performance turbo-machinery applications.

Keywords: *numerical acoustic, scattering, rotating frames,*

1. INTRODUCTION

The work presented here deals with the problem of efficient sound propagation modeling in ducted systems, particularly in the presence of fast-rotating domains. An example is the calculation of acoustic transmission losses through combustion engine intake/exhaust lines. Modern intake/exhaust systems normally contain turbo-compressors between the engine and the intake/exhaust, which can either block or transmit engine noise. Moreover, these compressors mainly operate in the low-frequency range for which the acoustic installation effect can be significant. Therefore, the radiated noise of those compressors can be strongly modulated by the system's acoustic response.

Sound scattering through small automotive compressors is often computed with low-dimensional (0D, 1D) models [1–3]. However, the results of these models are only accurate at very low frequencies below 30% of the first cut-on frequencies and for low rotational speeds.

Here, we propose to use a three-dimensional model based on a stationary and harmonic form of the linearized equations of fluid flow to compute acoustic scattering through rotating domains. In general, linearized equations have been used in many previous studies to compute sound scattering and acoustic instabilities in quiescent geometries [4–7]. As an extension of previous studies and to include the rotation of the compressor, we add a rotating reference frame. This accounts for the advection of mass, momentum, and entropy by additional rotation-related terms in the linearized equations. We demonstrate that for a rotor spinning at 96,000 RPM, this addition improves computational results compared to measurements on a similar geometry.

2. METHOD

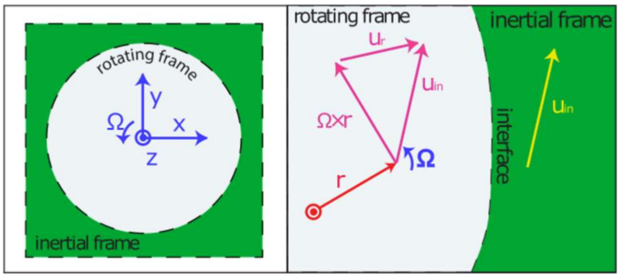

Figure 1. The rotating reference frame boxed into the inertial frame (left). Velocities in the rotating frame and in the inertial frame (right).

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To derive the acoustic conservation equations in a rotating frame of reference, the following steps are performed: First, the equations of fluid flow are expressed in rotating variables and temporal derivatives in the rotating frame [8]

$$[\tfrac{\partial\rho}{\partial t}]_r + \nabla\cdot\rho\mathbf{u_r} = 0 \tag{1}$$

$$\rho\left[\frac{\partial\mathbf{u_r}}{\partial t}\right]_r + \rho\mathbf{u_r}\cdot\nabla\mathbf{u_r} = \nabla\cdot\boldsymbol{\sigma}_\mathbf{r} +$$

$$\mathbf{F_r}\ \underbrace{-2\rho\boldsymbol{\Omega}\times\mathbf{u_r}}_{Coriolis\ force}\underbrace{-\rho\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r})}_{Centrifugal\ force} \tag{2}$$

$$\rho T\left[\frac{\partial s}{\partial t}\right]_r + \rho T\mathbf{u_r}\cdot\nabla s = \nabla\cdot k\nabla T + \emptyset_r + S_h \tag{3}$$

and $\rho T\mathrm{d}s = \rho c_p\mathrm{d}T - \alpha_p T\mathrm{d}p,$

with the density ρ, the pressure p, the velocity $\mathbf{u} = [u_x\ u_y\ u_z]^\mathrm{T}$, the entropy s, the temperature T, the viscous dissipation function $\emptyset_r = \nabla\mathbf{u_r}\colon\boldsymbol{\tau}_\mathbf{r}$, the viscous stress tensor $\boldsymbol{\tau}$, the total stress tensor $\boldsymbol{\sigma}$, the coefficient of thermal expansion α_p, the specific heat capacity at constant pressure c_p, and the thermal conductivity k. $\mathbf{F}_\mathrm{in}$ and S_h are source terms for the momentum and energy equation, respectively. The index $(\dots)_r$ indicates quantities with respect to the rotating frame and $[\frac{\partial}{\partial t}]_r$ is the temporal derivative in rotating coordinates. The operator ":" means a double contraction of the tensor product.

Second, using the relation between internal and rotating velocity (see Fig. 1), i.e. $\mathbf{u_r} = \mathbf{u_{in}} - \boldsymbol{\Omega}\times\mathbf{r},$ we can formulate Equ. 1-3 in terms of inertial velocity and time derivatives in the rotating frame. This formulation is advantageous, as it allows us to box the rotating frame into a stationary geometry, and solve for the acoustic state variables in terms of inertial coordinates

$$\left[\frac{\partial\rho}{\partial t}\right]_r + \nabla\cdot\rho\mathbf{u_{in}}\quad\underbrace{-(\boldsymbol{\Omega}\times\mathbf{r})\cdot\nabla\rho}_{rot.\ advection\ of\ density} = 0 \tag{4}$$

$$\rho\left[\frac{\partial\mathbf{u_{in}}}{\partial t}\right]_r + \rho\mathbf{u_{in}}\cdot\nabla\mathbf{u_{in}}\quad\underbrace{-\rho(\boldsymbol{\Omega}\times\mathbf{r})\cdot\nabla\mathbf{u_{in}}}_{rot.\ advection\ of\ momentum} =$$

$$\nabla\cdot\boldsymbol{\sigma}_\mathbf{in} + \mathbf{F_r}\quad\underbrace{-\rho\boldsymbol{\Omega}\times\mathbf{u_{in}}}_{Coriolis\ force\ contribution} \tag{5}$$

$$\rho T\left[\frac{\partial s}{\partial t}\right]_r + \rho T\mathbf{u_{in}}\cdot\nabla s\quad\underbrace{-T\rho(\boldsymbol{\Omega}\times\mathbf{r})\cdot\nabla s}_{rot.\ advection\ of\ entropy} =$$

$$\nabla\cdot k\nabla\mathrm{T} + \emptyset_{in} + S_h. \tag{6}$$

The transformation gives rise to additional advection terms in the equation of motion, caused by the rotation of the domain. In addition, a Coriolis force-related term appears in the momentum equation (Equ. 5). If the rotational speed of the domain is high, these terms can alter the scattering of sound through the domain.

Third, Equations 4-6 can be linearized to form the acoustic equations in a rotating reference frame. Additionally, a time-harmonic Ansatz was used for the acoustic quantities, transforming the equations into the frequency domain:

$$p' = \hat{p}e^{i\omega t}, \tag{7a}$$
$$\rho' = \hat{\rho}e^{i\omega t}, \tag{7b}$$
$$T' = \hat{T}e^{i\omega t}, \tag{7c}$$
$$\boldsymbol{u}' = \hat{\boldsymbol{u}}e^{i\omega t}, \tag{7d}$$

Where $(\dots)'$ denotes the time-harmonic acoustic perturbation, $\widehat{(\dots)}$ is the amplitude, and ω is the angular frequency. A detailed derivation of the linear, harmonic form of Equations 4-6 can be found in Ref. [8]. The linearized equations can be solved numerically. A simple and straightforward approach is to add the terms related to rotation as sources to existing FEM implementations of the Linearized Navier-Stokes or the Linearized Euler equation. This was done in the present study, where the equations were implemented in the commercial FEM solver COMSOL Multiphysics.

3. RESULTS

Figure 2 shows the results from the numerical computations using a rotating frame of reference. In Figure 2a, the discretized domain is presented. The green-colored part represents the domain in which a rotating frame of reference is used, while the gray-colored part represents the domain in which the inertial frame of reference is used. Note that the visibility of the inlet and the compressor volute was disabled in the plot. The numerical computation involved two steps:

1. The background mean flow at an operating point was computed by solving the Reynolds Averaged Navier Stokes equation using the commercial solver Star CCM+.

2. The acoustic field was computed with the Linearized Navier Stokes Equations in the frequency domain using the commercial solver Comsol Multiphysics, using the results from the first step as a background mean flow. Planar acoustic waves were excited at the inlet of the compressor, and the acoustic field was sampled at the outlet of the compressor.

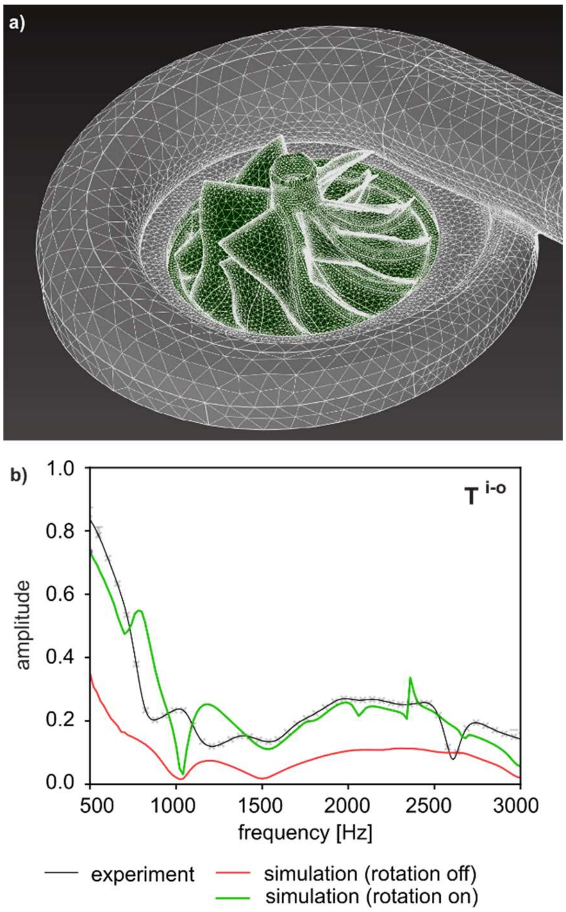

Figure 2. a) FEM model of an industrial turbo-compressor with inertial frame (grey) and rotating frame of reference (green). b) Sound wave transmission from intake to outlet, from experiments (black), and numeric solutions of the linearized Navier Stokes without rotational frame of reference (red) and with rotational frame of reference (green). Results are shown for a rotational speed of 96000 RPM.

The inlet and outlet pressure values were used to compute the transmission coefficient that is plotted in Figure 2b. The black line shows an aeroacoustics measurement on a geometrically identical compressor for the same operating point. The red line is the solution of the classical Linearized Navier Stokes Equations without a rotating frame of reference. Although the shape of the transmission coefficient was predicted well, the level of the transmission coefficient was underestimated. In contrast, the Linearized Navier Stokes Equations that included the rotating frame of reference estimated both the level and the shape much better.

4. SUMMARY

We gave an overview of a new method to include rotational effects into the classical formulations of the linearized aeroacoustic equations. We propose to use a rotating frame of reference, for which the acoustic state variables are expressed in terms of inertial coordinates and rotating time derivatives. This allows us to box rotational domains into existing geometries. The computation uses a stationary discretization mesh and includes rotation in form of additional advective terms. We show that the effect of rotation becomes important when computing acoustic scattering through fast rotating domains, e.g. automotive turbo-compressors. For more details and results please refer to Ref. [8].

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