SOUND ABSORPTION OF MULTISCALE SORPTIVE METAMATERIALS

R. Venegas[1*] **G. Núñez**[1] **J. Rebolledo**[2]
J. P. Arenas[1] **F. Orellana**[1] **C. Boutin**[3]
[1] Institute of Acoustics, University Austral of Chile, Chile
[2] Institute of Materials and Thermo-mechanical Processes, University Austral of Chile, Chile
[3] LGCB / LTDS UMR-CNRS 5513 / CeLyA, Université de Lyon - ENTPE, France

ABSTRACT

This work investigates the sound absorption properties of permeable metamaterials which include acoustic resonators and multiscale sorptive porous media as key building-block elements. The macroscopic description of acoustic wave propagation in multiscale sorptive metamaterials is established using the two-scale method of homogenisation. This upscaling procedure elucidates the key role of pressure-activated mass sources on the atypical effective compressibility of the studied class of metamaterials. The results are exemplified by introducing analytical models which are successfully compared with numerical predictions. Furthermore, particular attention is paid on to how inner resonances and diffusion processes affected by nanoscale phenomena can be exploited to engineer efficient low-frequency sound absorbers.

Keywords: *Metamaterials, sorptive media, sound absorption, inner sources, homogenisation*

1. INTRODUCTION

Gas-saturated multiscale sorptive porous materials [1–4] exhibit distinctive characteristic lengths that can range from nanometres up to millimetres or larger. In these materials, if the mass fluxes in the fluid networks are highly contrasted, the long acoustic wave is gradually carried by the more permeable networks, while the less permeable networks experience local dynamics in response to the uniform pressure in the more permeable fluid networks. The local dynamics is determined by either pressure or mass diffusion depending on the characteristic size of the smaller scale, and both phenomena are influenced by sorption effects in the nanopores of the material. Consequently, the diffusion phenomena play a significant role in determining the unconventional behaviour of the effective compressibility of multiscale sorptive porous media. Furthermore, the combination of classical visco-thermal dissipation and the diffusion phenomena affected by sorption leads to a significant dissipation of sound energy in broad frequency bands centred around the characteristic frequencies associated with the dissipation mechanisms. This dissipation of sound energy is the physical origin of the remarkable low-frequency sound-absorbing properties of multiscale sorptive media, as demonstrated theoretically and experimentally in previous studies [1–4].

On the other hand, acoustic metamaterials have gained significant attention in recent decades [5–8] because of their unusual effective acoustic properties, typically induced by their building-block elements' resonating nature. Directly relevant to this study is the work by Boutin et al. [5, 6] on the application of the two-scale asymptotic method of homogenisation to the upscaling of sound wave propagation in arrays of acoustic resonators surrounded by a pore fluid network or embedded in a porous matrix. These works are here extended to account for additional scales of porosity and nano-scale phenomena. As the non-conventional acoustic behaviour of such metamaterials depends on the resonant behaviour of the local constituents, their atypical properties are usually confined to narrow frequency bands.

This paper investigates the sound absorption behaviour of a novel class of metamaterials denominated multiscale sorptive metamaterials. These materials are

**Corresponding author*: *rodolfo.venegas@uach.cl.*

a combination of locally-resonant acoustic metamaterials and multiscale sorptive materials. The role of pressure-activated inner sources on the atypical effective compressibility of the studied class of materials is highlighted. The peculiar behaviour of the effective compressibility of the materials will be shown to lead to enhanced low-frequency sound absorption. An analytical model that is successfully compared with the sound absorption coefficient predictions obtained from direct local-scale numerical simulations is also introduced.

2. THEORY

2.1 Geometry

Figure 1 shows a diagram of the scales of a rigid-frame multiscale sorptive metamaterial saturated with a Newtonian fluid. The material's representative elementary volume (REV) Ω comprises a pore fluid network Ω_f, a Helmholtz resonator Ω_r, and a permeable heterogeneity Ω_p made of a multiscale sorptive material exhibiting pores or inclusions with characteristic sizes ranging from nm to mm [1–4]. The boundaries Γ_s, Γ_p, Γ_r are perfectly impervious walls, the interface $\partial\Omega_p \cap \partial\Omega_f$, and the interface between the resonator's neck and the pore fluid network, respectively. Volume fractions $\varphi_p = \Omega_p/\Omega$ and $\varphi_r = \Omega_r/\Omega$ will be used in the developments. The porosity of the material is ϕ. The period of the material is l and a long-wavelength regime is assumed, meaning that $\varepsilon = l/L \ll 1$ with $L = \lambda/2\pi$ being the macroscopic characteristic length L and λ the sound wavelength.

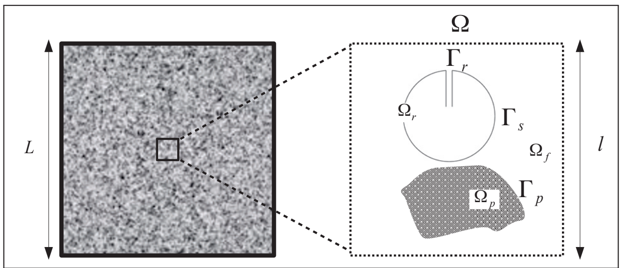

Figure 1. Diagram of the scales of a multiscale sorptive metamaterial.

2.2 Outline of the homogenisation process

This section outlines the application of the two-scale asymptotic method of homogenisation [10] to upscale the response of a multiscale sorptive metamaterial to acoustic waves. At the local scale, this response is determined by the Fourier-Stokes system (see, e.g., equations 1–4 in [4]) formulated in the pore fluid network Ω_f and the equations of conservation of mass and dynamic Darcy's law (see, e.g., equations 5 and 6 in [4]) formulated in Ω_p, i.e. in the local heterogeneity made of an isotropic multiscale sorptive material having an effective compressibility and dynamic viscous permeability being respectively denoted by C_p and $\mathcal{K}_p$. Continuity of mass flux, pressure, and excess temperature are formulated on both Γ_r and Γ_p, while zero velocity and excess temperature are set on Γ_s.

The physical analysis of the Fourier-Stokes system is classical (see, e.g. [10]). Regarding the equations formulated in Ω_p, one has that the Darcy velocity and pressure vary locally [4]. Moreover, the mass flux pulsed from the sorptive material is an order smaller than the mass flux generated by the sound wave propagating in Ω_f. This is valid in high-permeability contrast condition which occurs when $\ell/l = O(\varepsilon)$, where ℓ is the local characteristic size of the largest-pore scale of the sorptive material. Similarly, the mass flux pulsed from the acoustic resonator is also an order smaller than the mass flux generated by the sound wave propagating in Ω_f, which is a valid approximation if a characteristic size of the resonator (e.g., the radius of its neck) is one order smaller than l (see, e.g., [4]).

To account for variations at the scale of observation (i.e. the macroscopic scale) and at the local scale, two independent spatial variables are introduced, namely x and $y = \varepsilon^{-1}x$. Then, the unknowns become a function of these variables and $\nabla \rightarrow \nabla_x + \varepsilon^{-1}\nabla_y$. Using the results of the physical analysis and one of the introduced spatial variables, the rescaling of the local governing equations is performed (see, e.g., [10]). The rescaled Fourier-Stokes system formulated in Ω_f is shown by equations A1–4 in [9] while the rescaled equations in Ω_p are those shown in equation 16 in [3]. The rescaled boundary conditions on Γ_r are the equations A7–A9 in [4], while the boundary conditions on Γ_p are equations 17–19 in [3].

The variables are written as expansion series in terms of the small parameter ε and are then inserted into the rescaled equations. It then follows the collection of terms with equal powers of ε. At ε^{-1}, it is is found that $\nabla_y p^{(0)} = \mathbf{0}$, which means that $p^{(0)} = p^{(0)}(x) = P$ is a macroscopic variable. The identified boundary-value problems are: i) an oscillatory Stokes problem (formulated in Ω_f) forced by the macroscopic pressure gradient (see, e.g., equations in A15-19 in [9]), ii) an oscillatory heat conduction problem (formulated in Ω_f) forced by the macroscopic pressure (see, e.g., equations A22-26 in [9]),

iii) a pressure diffusion problem (formulated in Ω_p) forced by the macroscopic pressure on Γ_p (see, e.g., equations B4-5 in [4]). The solution of these problems are given, respectively, by equations A21 and A27 in [9], and equation B6 in [4], respectively. By spatially averaging the leading-order equation of conservation of mass, one obtains

$$\nabla_x \cdot \mathbf{V} = -\mathrm{j}\omega(\mathcal{S}_f + \mathcal{S}_p + \mathcal{S}_r), \tag{1}$$

where $\mathbf{V}$ is the oscillatory Darcy's velocity and the effective pressure-activated volumetric sources are given by

$$\mathcal{S}_f = \mathsf{C}_f P \quad ; \quad \mathcal{S}_p = \varphi_p \mathsf{C}_p \mathcal{F}_p P \quad ; \quad \mathcal{S}_r = \varphi_r \tilde{\mathsf{C}}_r P. \tag{2}$$

Here, C_f is the effective compressibility of the pore space (given by equation 23 in [4]), C_p is the effective compressibility of the multiscale sorptive material (given by equation 22 in [4]), $\mathcal{F}_p$ is the spatially averaged ratio between the acoustic pressure in the sorptive material and that in the pore fluid network (see equation 24 in [4]), and $\tilde{\mathsf{C}}_r$ is an apparent compressibility, given by equation 28 in [9], which depends on the effective admittance of the resonator $\mathcal{Y}_r$ via $\tilde{\mathsf{C}}_r = 2\mathcal{Y}_r / j\omega\mathcal{L}_r$ where $\mathcal{L}_r = 2\Gamma_r/\Omega_r$. Eq. (1) and the dynamic Darcy's law (with η being the dynamic viscosity of the saturating fluid), i.e.,

$$\nabla_x \cdot \mathbf{V} = -\frac{\mathbf{k}}{\eta} \cdot \nabla_x P, \tag{3}$$

correspond to the macroscopic description of sound propagation in multiscale sorptive metamaterials with pore/sorptive material high permeability contrast.

For the case of pore/sorptive material low permeability contrast, the effective parameters are given, in a first approximation, by

$$\mathbf{k} = \mathbf{k}_f + \varphi_p \mathbf{k}_p \quad \text{and} \quad \mathsf{C} = \mathsf{C}_f + \varphi_p \mathsf{C}_p + \varphi_r \tilde{\mathsf{C}}_r. \tag{4}$$

This is an exact formulation for materials with pores Ω_f having a constant cross-section, as it is the case of the example presented below. It is emphasised that inner volumetric sources, whose physical origin is heat conduction at different scales, and pressure and mass diffusion, determine the acoustic behaviour of C_p [4]. Thus, several inner volumetric sources characterise the atypical acoustic behaviour of the effective compressibility of the multiscale sorptive metamaterials investigated in this work.

3. RESULTS

Figure 2 shows the geometry of an air-saturated multiscale sorptive metamaterial. Its representative elementary volume (REV) comprises a mesoperforated triple porosity granular sorptive matrix, a Helmholtz resonator, and an annular pore formed in between the resonator and the matrix. The dimensions of the metamaterial are $w_o = 5.0$ cm, $w_i = 4.2$ cm, $d = 4.2$ cm, $w = 5 \times \sqrt{\pi}$ cm, $a_r = 1$ mm, $l_r = 16$ mm (see Fig. 2). The matrix's material is that investigated in [4] (see table IV for its parameters). Hence a low-permeability contrast condition between the pore fluid network and the matrix is identified. The dynamic viscous permeability depends on $\mathcal{K}_f$ and $\mathcal{K}_p$, which correspond to the dynamic viscous permeability of an array of annular pores (see equations 30 and 33 in [11]) and of the multiscale sorptive material (see table II in [4]), respectively. The effective compressibility depends on C_f, C_p, and $\tilde{\mathsf{C}}_r$, which are respectively given by the inverse of the equation 35 in [11], the equations shown in the table I in [4], and the equations 28 and B2 in [9].

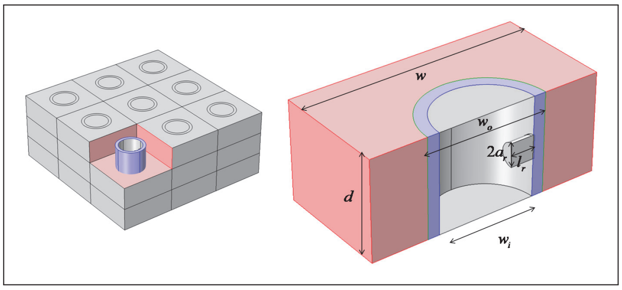

Figure 2. Geometry of a multiscale sorptive metamaterial with specific microstructure. The REV contains a mesoperforated triple porosity granular sorptive matrix, an annular pore, and a Helmholtz resonator.

Figure 3 shows the normalised effective compressibility of the multiscale sorptive metamaterial. A large value of the real part of this quantity is observed due to sorption effects in the nanopores of the matrix material [1–4]. A sharp transition in the real part of the effective compressibility occurs around the acoustic resonator's resonance frequency, together with both an increase in magnitude and the widening of the imaginary part. Specifically, the widening is caused by classical dissipation due to heat conduction as well as pressure and mass diffusion in the sorptive material. For higher frequencies, the effective compressibility of the metamaterial becomes closer to that of the matrix. This fact is because of the quasi-adiabatic nature of sound propagation in the building-block elements of the metamaterial. Thus, the atypical behaviour of the effective compressibility leads to a decrease in the speed of sound and an increase in the attenuation coefficient (not shown here for the sake of brevity).

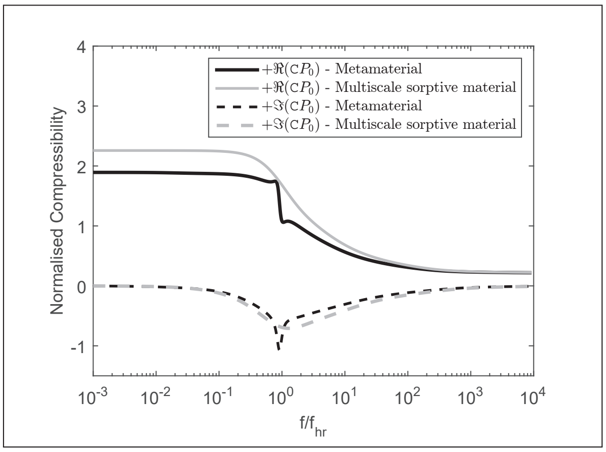

Figure 3. Normalised effective compressibility of a multiscale sorptive metamaterial as a function of the reduced frequency f/f_{hr}. The resonance frequency of the acoustic resonator is $f_{hr} = 108$ Hz

The sound absorption coefficient α of a finite rigidly-backed multiscale sorptive metamaterial layer of thickness d is now investigated. This is calculated analytically and numerically. For analytical calculations, α is determined from equation 29 in [4] using the effective parameters shown in Eq. (4). For the numerical calculations, a virtual impedance tube approach (see [12] for more details) is adopted. Specifically, the material layer is placed in a virtual impedance tube and the following equations are solved by using the finite element method: lossless Helmholtz equation (formulated in the upstream part of the tube), Helmholtz equation with effective wave number $k_c = \omega\sqrt{\eta \mathsf{C}_p / j\omega\mathcal{K}_p}$ (formulated in the sorptive material modelled as an effective fluid), and the Fourier-Stokes system (formulated in and around the acoustic resonators). Conditions of continuity of pressure and mass flux are considered on the material-fluid interfaces. In all perfectly-rigid boundaries, zero normal pressure gradient was set. The material is virtually excited with a plane wave. Figure 4 shows a comparison between α of rigidly-backed layers of both the matrix's material alone (i.e. a multiscale sorptive material) and the multiscale sorptive metamaterial. For the latter, a significant increase of α in the low-frequency region is observed. This is caused by the combined effect of the local resonances and the multiple dissipation mechanisms present in the matrix's material. Indeed, an α of approximately over 0.8 is observed from around 80 Hz onwards. Moreover, the analytical model for the multiscale sorptive metamaterial correctly captures the behaviour of α, as evidenced by the good agreement between its predictions and the direct pore-scale finite-element modelling (FEM) results obtained as explained above.

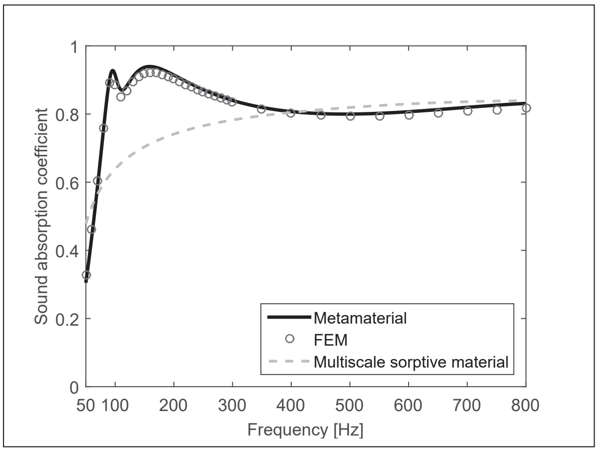

Figure 4. Sound absorption coefficient of rigidly-backed layers of the multiscale sorptive metamaterial and the matrix's material (i.e. a multiscale sorptive material) as a function of frequency. The layer thickness is $d = 12.6$ cm and $f_{hr} = 108$ Hz.

4. CONCLUSIONS

This paper investigated the sound absorption properties of a novel class of materials denominated multiscale sorptive metamaterials. The representative elementary volume of these air-saturated materials includes a multiscale sorptive material, a Helmholtz resonator, and a pore fluid network. The macroscopic description of acoustic wave propagation in the novel class of materials was established through the two-scale asymptotic homogenisation method. This elucidated the crucial role of pressure-activated inner mass sources on the atypical effective compressibility of the materials. In turn, such atypical behaviour explains the unusually high low-frequency sound absorption of the studied class of materials. This work sets the stage for further exploring the control of low-frequency acoustic waves by using multiscale physics phenomena and inner sources in acoustic metamaterials.

5. ACKNOWLEDGMENTS

This work was supported by the Chilean National Agency for Research and Development (ANID) through FONDECYT Regular Grant No. 1211310.

6. REFERENCES

[1] R. Venegas and O. Umnova. Acoustical properties of double porosity granular materials. *J. Acoust. Soc. Am.*, 130: 2765–2776, 2011.

[2] R. Venegas and O. Umnova. Influence of sorption on sound propagation in granular activated carbon. *J. Acoust. Soc. Am.*, 140: 755–766, 2016.

[3] R. Venegas and C. Boutin. Acoustics of sorptive porous materials. *Wave Motion*, 68: 162–181, 2017.

[4] R. Venegas, C. Boutin, and O. Umnova. Acoustics of multiscale sorptive porous materials. *Phys Fluids*, 29: 082006, 2017.

[5] C. Boutin. Acoustics of porous media with inner resonators. *J. Acoust. Soc. Am.* 134(6): 4717–4729, 2013.

[6] C. Boutin and F. X. Bécot. Theory and experiments on poro-acoustics with inner resonators. *Wave Motion* 54: 76–99, 2015.

[7] G. Ma and P. Sheng. Acoustic metamaterials : From local resonances to broad horizons. *Sci. Adv.*, 2 (2): e1501595, 2016.

[8] S. A. Cummer, J. Christensen, and A. Alù. Controlling sound with acoustic metamaterials. *Nav. Rev. Mater.*, 1: 16001, 2016.

[9] R. Venegas, G. Núñez, C. Boutin, O. Umnova, and Q. Zhang. Acoustic wave propagation in permeable lossy metamaterials. *Phys Fluids*, 34: 017117, 2022.

[10] J. L. Auriault, C. Boutin, and C. Geindreau. *Homogenization of Coupled Phenomena in Heterogeneous Media*. ISTE Ltd and John Wiley & Sons, 2009.

[11] M. Nori and R. Venegas. Sound propagation in porous materials with annular pores. *J. Acoust. Soc. Am.*, 141 (6): 4642–4651, 2017.

[12] T. G. Zielinski, R. Venegas, C. Perrot, M. Cervenka, F. Chevillotte and K. Attenborough, Benchmarks for microstructure-based modelling of sound absorbing rigid-frame porous media. *J. Sound Vib.*, 483: 115441, 2020.