MODELLING THE SOUND TRANSMISSION PROPERTIES OF A SIDE BRANCH ACOUSTIC METAMATERIAL USING RIGID FRAME POROUS MODEL

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ABSTRACT[*]

This paper presents a model to predict the sound transmission properties of a side branch acoustic metamaterial composed of coupled subwavelength Helmholtz resonators.

A finite element model (FEM) is proposed to describe the grazing incidence wave propagation in Helmholtz resonators. The Viscothermal losses due to the solid-fluid interaction on the boundary layer interface in the neck of resonators are considered by employing the classical fluid equivalent Johnson-Champoux-Allard (JCA) rigid frame approach. Next, the in-parallel coupling of multiple resonators is conducted to enlarge the frequency range.

To validate the results, an experimental campaign was developed based on the four-microphone measurement method (ASTM E2611). As expected, the preliminary numerical results show good agreement when compared with experimental values. In turn, the approach presented here enables the coupling of multiple resonances of individual resonators under grazing incidence, which can be effectively used in designing compact attenuators with a wide frequency range, with potential application in different engineering contexts, especially in ventilated systems.

Keywords: *Acoustic metamaterial, Helmholtz resonators, sound transmission properties, JCA rigid frame model***.**

1. INTRODUCTION

The propagation of low-frequency sound waves in constrained acoustic systems is a widely theorized field [1,2], with interest in the development of sound attenuation strategies in the diverse acoustic fields of activity. Indeed, specific restrictions exist in practice due to the characteristics of each application, e.g. heating, ventilation and air conditioning systems, automotive and aeronautical engineering [1].

Acoustic metamaterials with negative bulk modulus based on the classic theory of Helmholtz resonators (HR) have repeatedly been shown to allow significant sound transmission loss by inducing passive flow in a low-frequency regime with subwavelength dimensional structures [3]. Based on previously developed work [4], the present paper aims to assess and accurately model the sound transmission properties of an acoustic system composed by the in-parallel coupling of multiple subwavelength resonators and describe the grazing incidence wave propagation under a linear regime, by applying the Johnson-Champoux-Allard model for rigid frames [5,6] considering the estimations of transport parameters for rigid frame models.

The present work is organized as follows: In section 2.1, the modelling strategy is presented, and the geometrical description is discussed for the unit cell of the acoustic metamaterial (AMM). In section 2.2, the experimental

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procedure based on ASTM E2611-09 [7] is presented to describe the sound transmission properties of the proposed AMM. In section 2.3, the authors present the finite element approach describing the dissipative phenomena through the fluid equivalent Johnson-Champoux-Allard methodology for rigid frames [5,6]. Consequently, in Section 3, the comparison between the obtained results is presented. Finally, the main conclusions are presented and discussed in Section 4.

2. DESIGN OF THE HEXAGONAL ACOUSTIC METAMATERIAL

2.1 Hexagonal acoustic metamaterial - general description and sample fabrication

In this study, the authors propose an acoustic metamaterial based on the axial coupling of n Helmholtz resonators with distinct local resonances, for solving sound transmission problems in ventilated systems. In Figure 1 (a), $l_{\mathrm{n}}^{[n]}$ corresponds to the length of the resonator's neck, $w_{\mathrm{n},1}^{[n]}$ corresponds to the width of the neck aperture with a rectangular section, $l_{\mathrm{c}}^{[n]}$ corresponds to the length of the resonator cavity, $w_{\mathrm{c},1}^{[n]}$ and $w_{\mathrm{c},2}^{[n]}$ correspond, respectively, to the dimensions of the rectangular section cavity. For simplification purposes, the central hole corresponds to a circumscribed hexagon S_{h} with radius r_{h}.

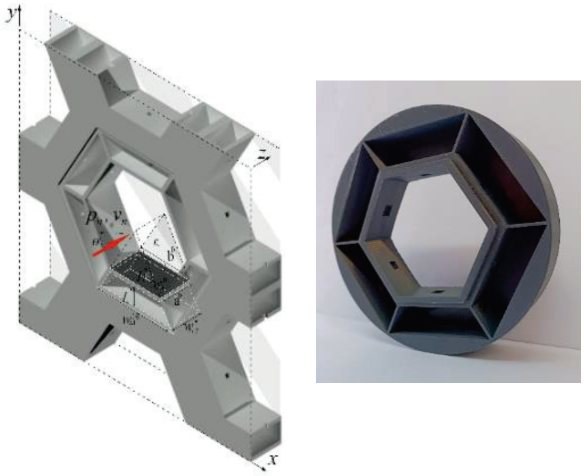


Figure 1. (a) Conceptual diagram of the unit cell composed of a set of n in-parallel Helmholtz Resonators coupled around each hole. (b) Photograph of a 3D printed AMM cell.

Considering the dimensions of a cylindrical impedance tube of 45 mm diameter, used in the experimental measurements described below, a cylindrical unit cell test sample was fabricated by using a Blocks One 3D printer with Polylactic Acid (PLA) and considering a nozzle extruder with 0.4 mm, to be experimentally evaluated according to the standardized method ASTM E2611-09[7], as described below. One of the produced samples is illustrated in Fig. 1 (b).

2.2 Experimental measurements

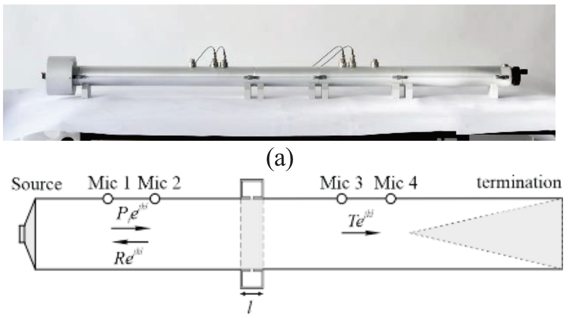
The estimation of the sound transmission loss (STL) was conducted by the four-microphone-method as described in ASTM E2611-09[7], where two measurements with different termination conditions perform the transfer matrix elements T_{11}, T_{12}, T_{21}, T_{22}, which can be used to determine the matrix T_{exp}, that can be written as follows:

$$T_{exp} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}. \tag{1}$$

Thus, the STL, expressed in decibels (dB), is calculated by:

$$\mathrm{TL} = 20\log\left(\frac{1}{2}\left|T_{11} + T_{12}/Z_0 + Z_0 T_{21} + T_{22}\right|\right). \tag{2}$$

The Fig. 2 illustrates the general setup, corresponding to an impedance tube with a diameter of d_w = 45 mm. The measurement frequency range of the impedance tube is specified as f_{i} = 50 Hz, the inferior limit, and f_{u} = 4150 Hz, the upper limit.



(a)

(b)

Figure 2. (a) Image and (b)schematic representation for the measurement setup.

2.3 Finite Element Modeling

To investigate the acoustic behaviour of the proposed AMM, a tridimensional (3D) finite element model (FEM) was implemented through the commercial software COMSOL Multiphysics, as illustrated in Fig. 3, numerically reproducing the standardized method for measurement of normal incidence sound transmission, described in the ASTM E2611-09[7]. In Fig. 3(b), the model assumes the periodicity condition imposed in a two-port model, under the excitation of an incident plane acoustic wave of unitary amplitude (1 Pa), on the left side of the model, with the waveguide consisting of a main cylindrical domain, with diameter d_w, with the waveguide and all surfaces being considered as perfectly rigid, and the perfectly matched layer (PML) being admitted at both ends of the waveguide, to mitigate subsequent reflections.

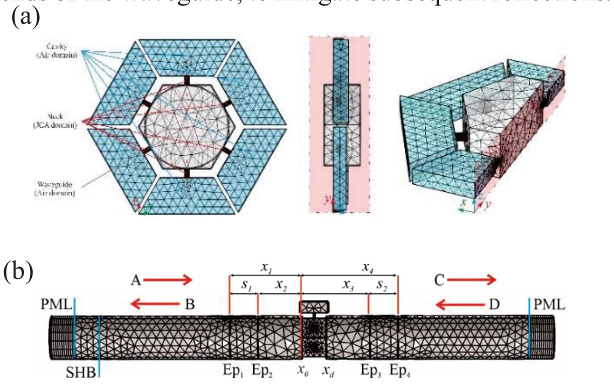


Figure 3. (a) FEM description of the proposed AMM unit cell model; (b) basic setup of the numerical model reproducing the ASTM E2611 (not to scale).

The discretization of the 3D FEM was performed as tetrahedral elements, with the maximum element size given by $\lambda_{min}/8$, where $\lambda_{min} = c_0/f_{max}$ is the smallest acoustic wavelength within the simulated range of frequencies, corresponding to a maximum frequency of 1 kHz. The narrow domains were modelled as 3D tetrahedral elements, and their discretization was performed in parts, as follows; in the necks of the HRs, the maximum mesh size is $w_n^{[n]}/4$ and the minimum size is $d_v/2$, where $d_v = \sqrt{2\mu/\rho_0\omega}$ is the thickness of the viscous boundary layer; by simplifications, in the cavities, considering the reduction of the dissipative effects inside them, the maximum mesh size is $w_c^{[n]}/2$ and $w_c^{[n]}/4$ as the minimum mesh sizes. Thus, comprising the adaptation of the JCA model for rigid media [5,6] the description of the viscous effect inside the inlet hole, as well as the acoustic fluid distortion in the outer rigid layer, is strictly controlled by its specific airflow resistivity, given by [6]:

$$\sigma = \left(\frac{2\,l_n}{r_h} + 4\right)\frac{R_S}{\phi}\frac{1}{l_n}; \tag{3}$$

being, ϕ, the porosity is equal to 1 [-]; the resistive term, R_s, is the surface resistance, which corresponds to the flux distortion dependent on the angular frequency ω, expressed by [6]:

$$R_s = \frac{\sqrt{2\eta\omega\rho_0}}{2}. \tag{4}$$

The distortion of the acoustic flow, due to the sudden change in cross-section associated with the inertial load due to sound radiation, expresses the reactive portion through the geometric tortuosity, $\alpha_\infty = 1$; the thermal and viscous characteristic lengths, Λ and Λ', are numerically equal to the hydraulic radius, $r_n = w_n/\sqrt{\pi}$.

3. PRELIMINARY RESULTS

3.1 Comparisons with the sound transmission loss measurements for single, dual and triple propositions

3.1.1 Single Frequency model

Initially, for the first case under analysis, the configuration of interest, consisting of a system composed by six identical Helmholtz resonators axially coupled and tuned, at $f_1 = 550$ Hz, Fig. 4 presents the corresponding comparison between the numerical prediction and experimental results for the STL.

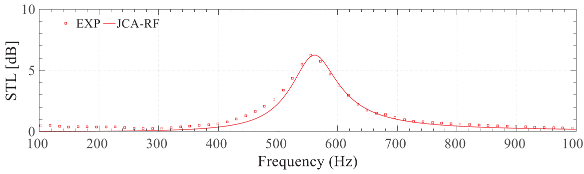


Figure 4. Sound transmission loss (STL) for the single resonant model, in decibels (dB).

The results of the sound transmission loss, for the single HR model, show a good agreement when compared with experimental measurements.

3.1.2 Multi-resonance model

In this topic, the authors describe the possibility of developing these structures through the coupling of resonators tuned to different resonances. The sound transmission properties of the dual case are presented, and the coupling of two sets of three identical resonators tuned to two resonance frequencies, at $f_1 = 560$ Hz and $f_2 = 625$ Hz, respectively. In Fig. 5, comparison of the measured results and numerical simulations is illustrated evidencing considerable agreement between both.

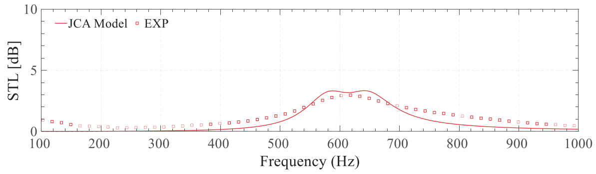


Figure 5. Sound transmission loss (STL) for the dual resonant model, in decibels (dB).

Finally, for the third case analyzed, in Fig. 6, the proposition of three resonance frequencies is presented. Being composed by the coupling of three sets of two resonators, tuned in three different resonance frequencies, at $f_1 = 540$ Hz, $f_2 = 590$ Hz and $f_3 = 740$ Hz, respectively.

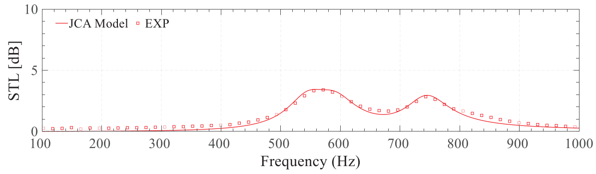


Figure 6. Sound transmission loss (STL) for the triple resonant model, in decibels (dB).

The results presented for the parallel arrangement of tuned resonators evidence the widening of the curve along the frequency range, conditioned by a significant reduction of the attenuation capacity of the system. In comparison with the experimental measurements presented, as previously reported, the JCA approach (red solid line) enabled by the FEM, ends up correctly accounting for the coupling of multiple resonance frequencies, thus ensuring good agreement when compared to experimental data.

4. CONCLUSIONS

In this work, the proposed AMM was evaluated in terms of sound transmission loss, and the influence of geometric parameters was observed for single, dual and triple resonance models. The models were validated through the FEM, applying the well-established JCA approach to describe the inherent dissipative losses originated in the solid-fluid interactions on boundary layers, showing good agreement between the presented results and the experimental measurements performed according to ASTM E2611-09[7]. It is thus expected that the present study can serve as an important contribution to the research and development of AMM with application in sound attenuator systems that enable airflow, in diverse engineering technical fields.

5. ACKNOWLEDGMENTS

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